



A market-oriented stochastic optimization framework and its application in the energy domain

Agenda

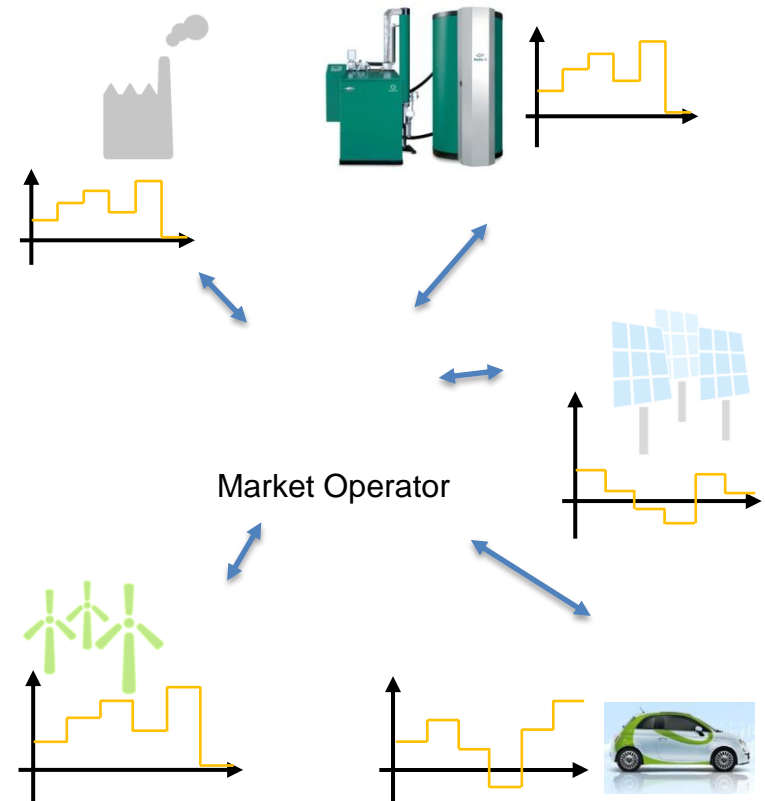
- Introduction
- Model and problem formulation
- Motivation for a stochastic market model
- Stochastic market optimization framework
- Preliminary simulation results

Introduction

- Motivation/ Assumptions
 - Increase in renewable energy generation
 - Increasing amount of DG and DRR
 - Trend towards smaller units
 - Trend towards locally distributed units

- Characteristics of DG and DRR
 - Individual cost function
 - Local device restrictions
 - Represented by a software agent that participates in the market system

- Problem domain:
 - Optimization of unit commitment
 - **Goal:** Increase overall welfare by reducing the energy generation costs



Model and problem formulation

- Each prosumer agent $i \in I := \{i \mid 1 \leq i \leq n, i \in \mathbb{N}\}$ has a ...
 - Preferred energy allocation vector $x = (x[1], \dots, x[k])^T$
 - $x[j] > 0$: consumption in period j
 - $x[j] \leq 0$: production in period j
 - Individual cost function $\text{cost}(\cdot) : \mathbb{R}^k \mapsto \mathbb{R}$
 - Local device restrictions $\mathfrak{R} \subseteq \mathbb{R}^k$
 - Typical values: $k = 96, n \geq 1\text{Mio.}$

- Definition **Unit Commitment Problem (UCP)**: The UCP comprises the task of computing $n = |I|$ energy schedules $x_i \in \mathbb{R}^k$, such that

$$\begin{aligned} & \underset{x_i \in \mathfrak{R}_i}{\text{argmin}} && \sum_{i \in I} \text{cost}_i(x_i) \\ & \text{s.t.} && \sum_{i \in I} x_i[j] = 0; \quad \forall j : 1 \leq j \leq k \end{aligned}$$

Lagrange Relaxation → General Equilibrium Problem

- Lagrange Relaxation

$$L_{\text{Dual}} := \max_{\lambda \in \mathbb{R}^k} \left\{ L(\lambda) := \operatorname{argmin}_{x_i \in \mathfrak{X}^k} \left\{ \sum_{i \in I} \operatorname{cost}_i(x_i) - \sum_{i \in I} \lambda \cdot x_i \right\} \right\} \quad \text{Lagrange Multiplier}$$

separable

- Note: $\max_{\lambda \in \mathbb{R}^k} \Leftrightarrow \partial L / \partial \lambda = 0 \Leftrightarrow \sum_{i \in I} x_i = 0$

- Definition **General Equilibrium Problem (GEP)**: *The GEP comprises the task of finding a price vector $p = (p_1, \dots, p_k)$ such that:*

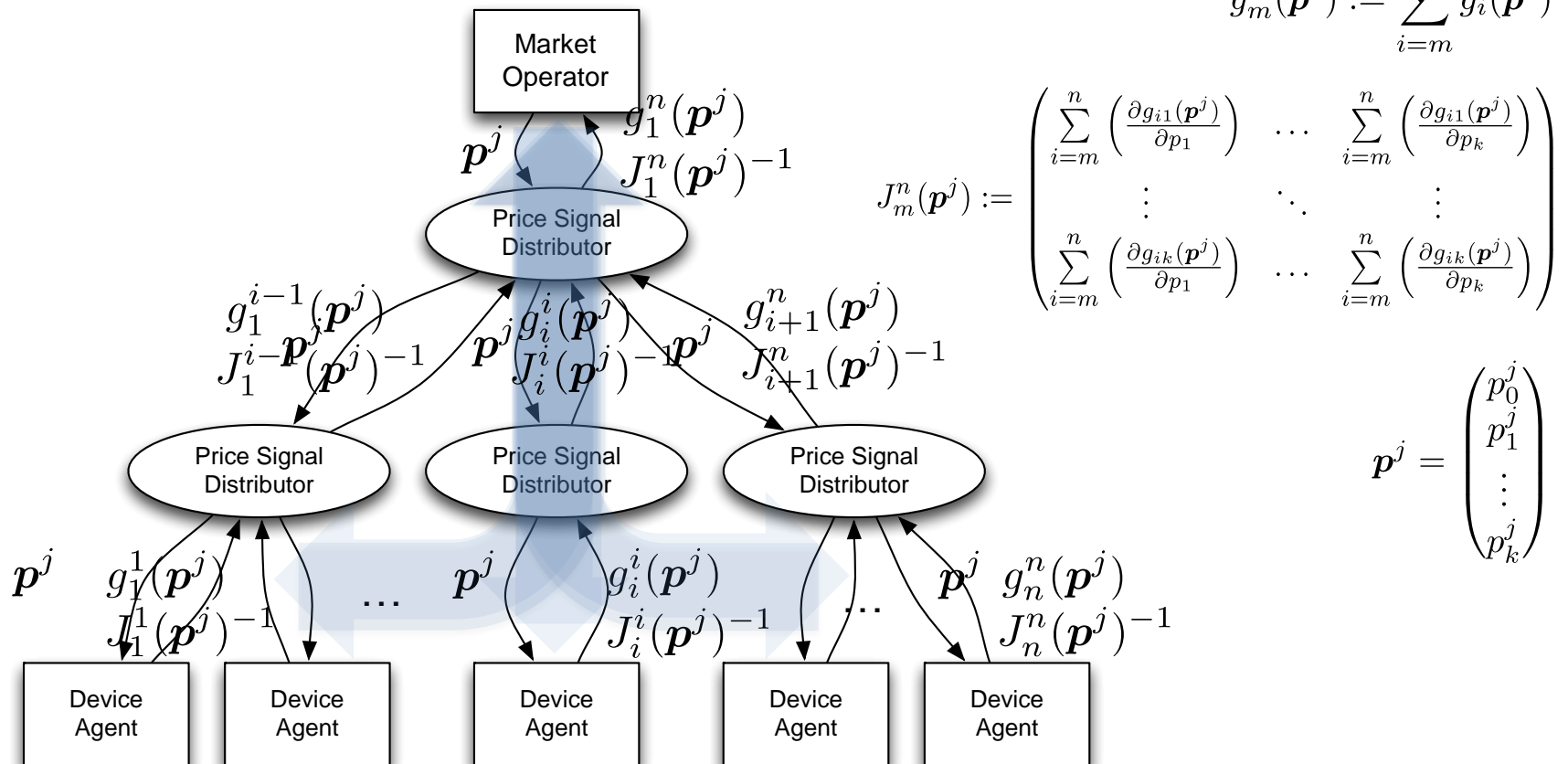
$$\sum_{i \in I} g_i(p) = 0 \quad \text{Root finding of } k \text{ non-linear equations}$$

$$g_i(p) := \operatorname{argmin}_{x_i} \operatorname{cost}(x_i) + p \cdot x_i \quad \forall i \in I$$

Newton Raphson solution/ Hierarchical information flow

$$\mathbf{p}^{j+1} = \mathbf{p}^j - J_1^n(\mathbf{p}^j)^{-1} \cdot g_1^n(\mathbf{p}^j)$$

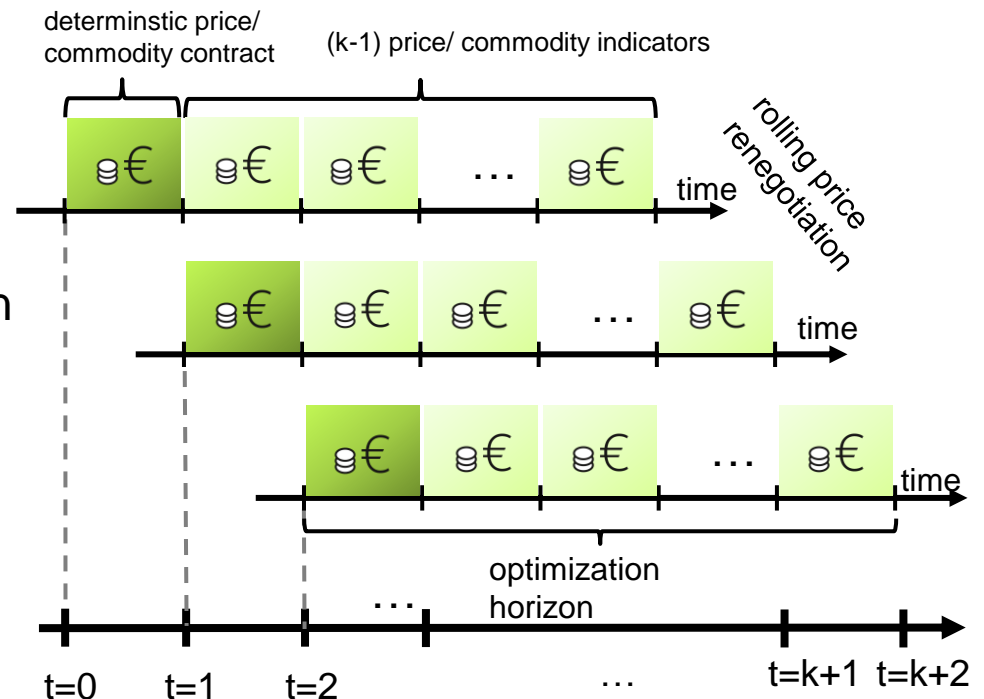
$$g_m^n(\mathbf{p}^j) := \sum_{i=m}^n g_i(\mathbf{p}^j)$$



Summary deterministic market model

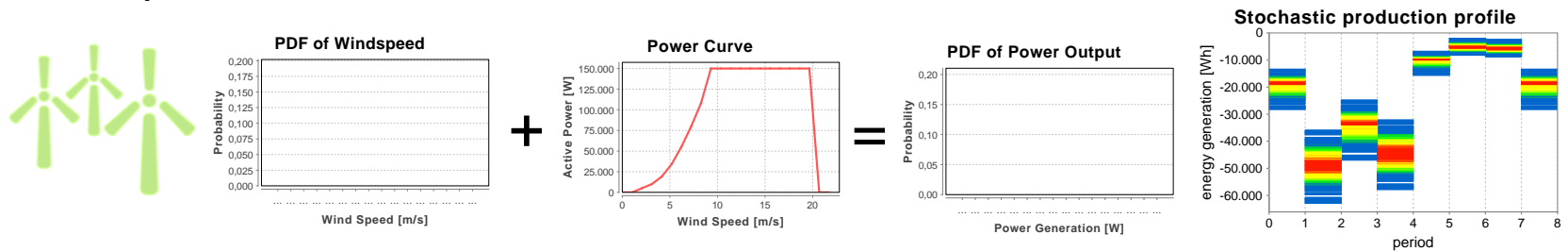
- Fix optimization horizon (typically 96 - 1/4h periods)
- (De)centralized deterministic market model → iterative NR based price protocol determines market clearing price

- Autonomous software agents negotiate energy schedules
- Each unit has an individual gain function $g_i(\cdot) : \mathbb{R}^k \mapsto \mathbb{R}^k$ that models the reaction on the price signal
- Rolling horizon implementation

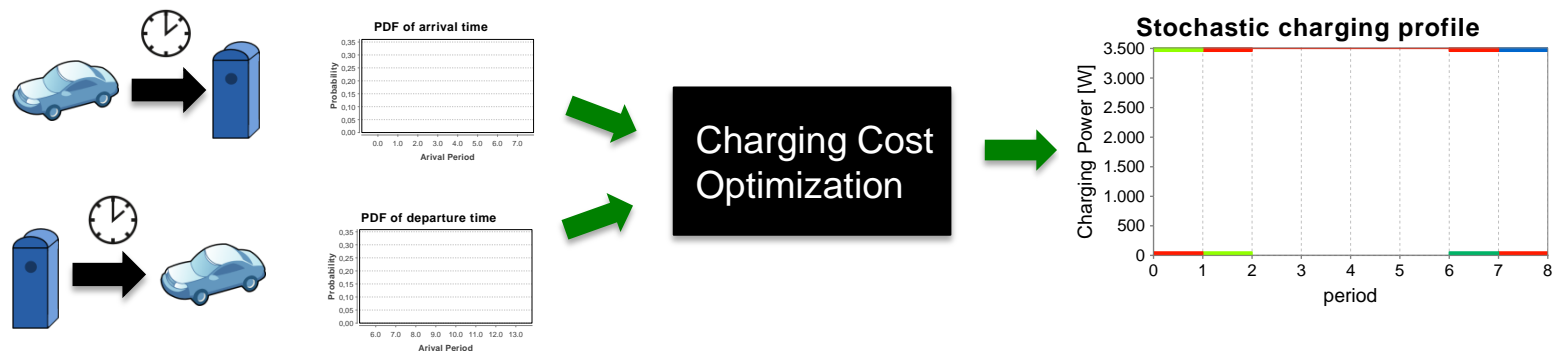


Motivation stochastic market framework

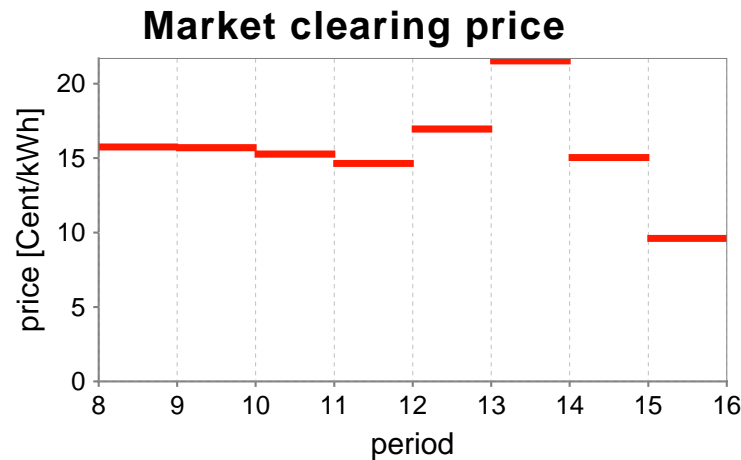
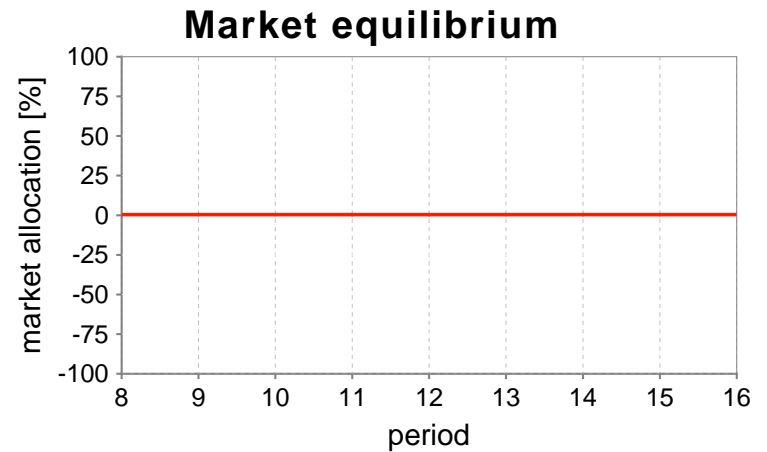
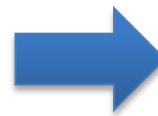
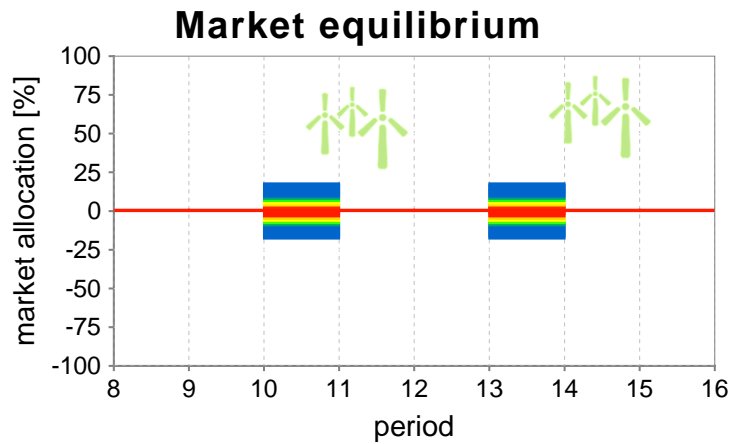
- An increasing amount of market participants have stochastic consumption/ production profiles:
- Example wind farm:



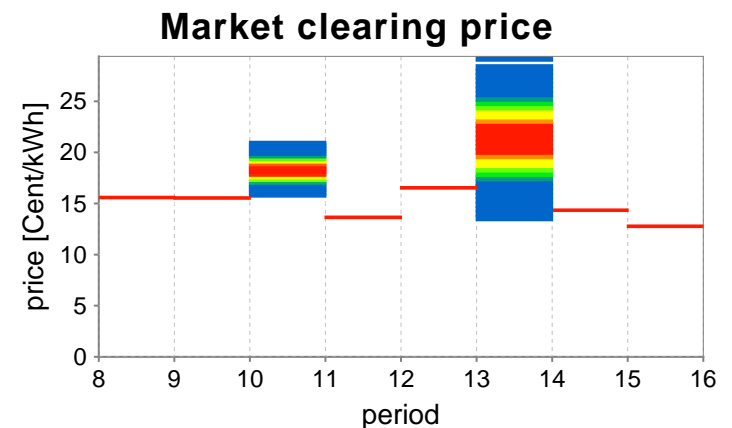
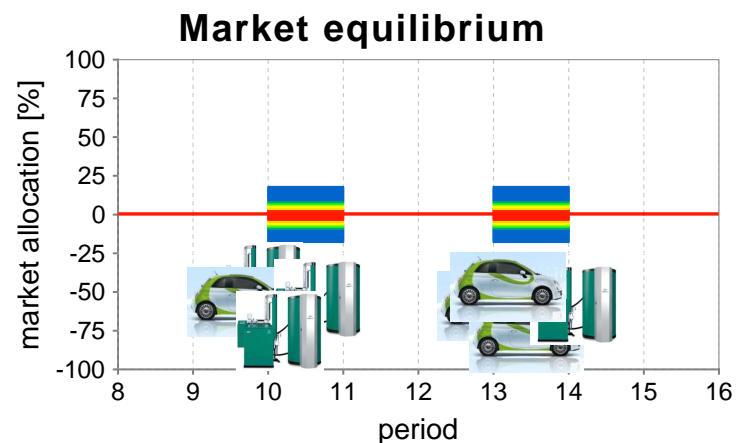
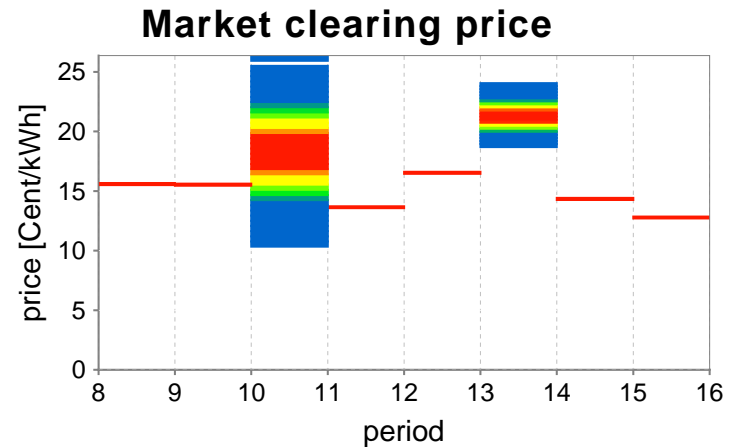
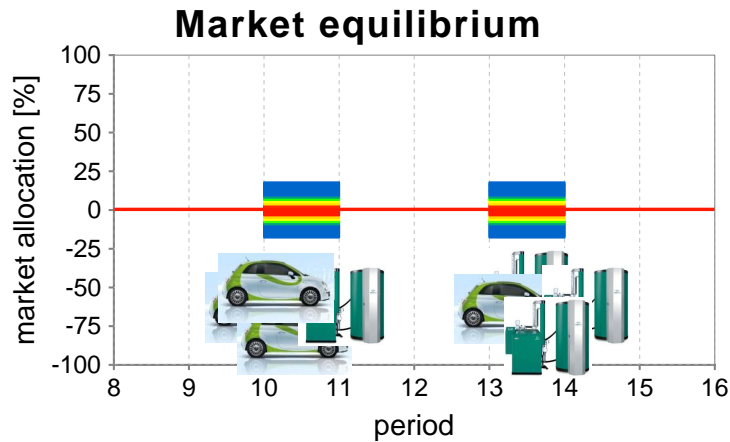
- Example electric vehicle as DRR:



Insufficient incentives



New incentives for flexibilities

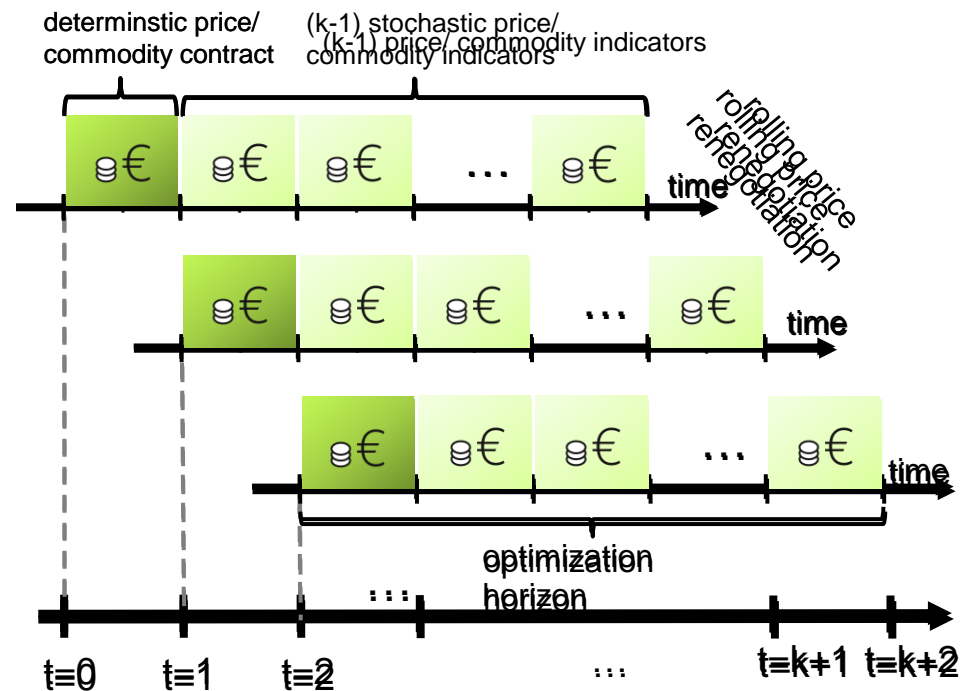


Stochastic multi commodity market model

Extended Model

- Stochastic commodity allocations (PDF for each commodity)
 - Commodity uncertainties are mapped to price uncertainties

- Assumptions:
 - Stochastic independence among multiple consumer/producer
 - No intermediate forecast updates



Problem formulation

- Definition **Stochastic General Equilibrium Problem (SGEP)**: *The SGEP comprises the task of finding a stochastic price vector $\mathbf{P} = (P_1, \dots, P_k)$ where $P_i : \Omega \rightarrow \mathbb{R}$ are Random Variables such that:*

$$\mathbb{E} \left[\sum_{i=1}^n g_i(\mathbf{P}) \right] = 0$$

Local stochastic optimization

$$g_i(\mathbf{P}) := \max_{\mathbf{X}_i} \text{cost}(\mathbf{X}_i) + \mathbb{E}[\mathbf{P}] \cdot \mathbb{E}[\mathbf{X}_i]$$

$g_i(\boldsymbol{\rho}) = (X_{i1} = g_{i1}(\mathbf{P}), X_{i2} = g_{i2}(\mathbf{P}), \dots, X_{ik} = g_{ik}(\mathbf{P}))$ denotes the stochastic commodity vector. $X_{ij} : \Omega \rightarrow \mathbb{R}$ are Random Variables which indicate the stochastic demand / supply of the i -th market participant (Agent) in the j -th period.

Mapping commodity uncertainties to price domain

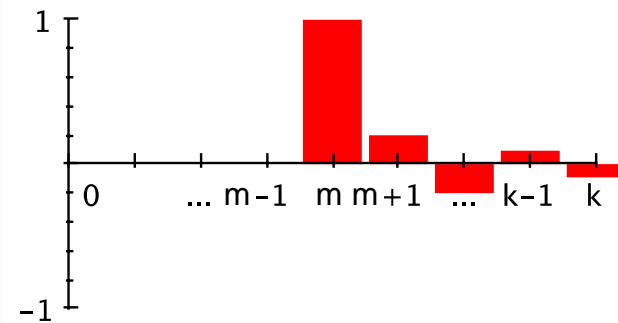
- How can we approximate the influence of an uncertain allocation deviation in period m on market prices?
 - We may sample the rebalancing response on a given price incentive.
- Definition Rebalancing Price Response (RePR):** For each market participant i , the constant price deviation Δp in period m of the current stochastic price vector P and the existing commodity allocation $X_i = g_i(P)$ the Rebalancing Price Response $g_{\text{RPR},i}$ is defined as:

Market participant

$$g_{\text{RPR},i}(P, m, \Delta p) := \max_{X'_i} \text{cost}_i(X'_i) + X'_i \cdot P'$$

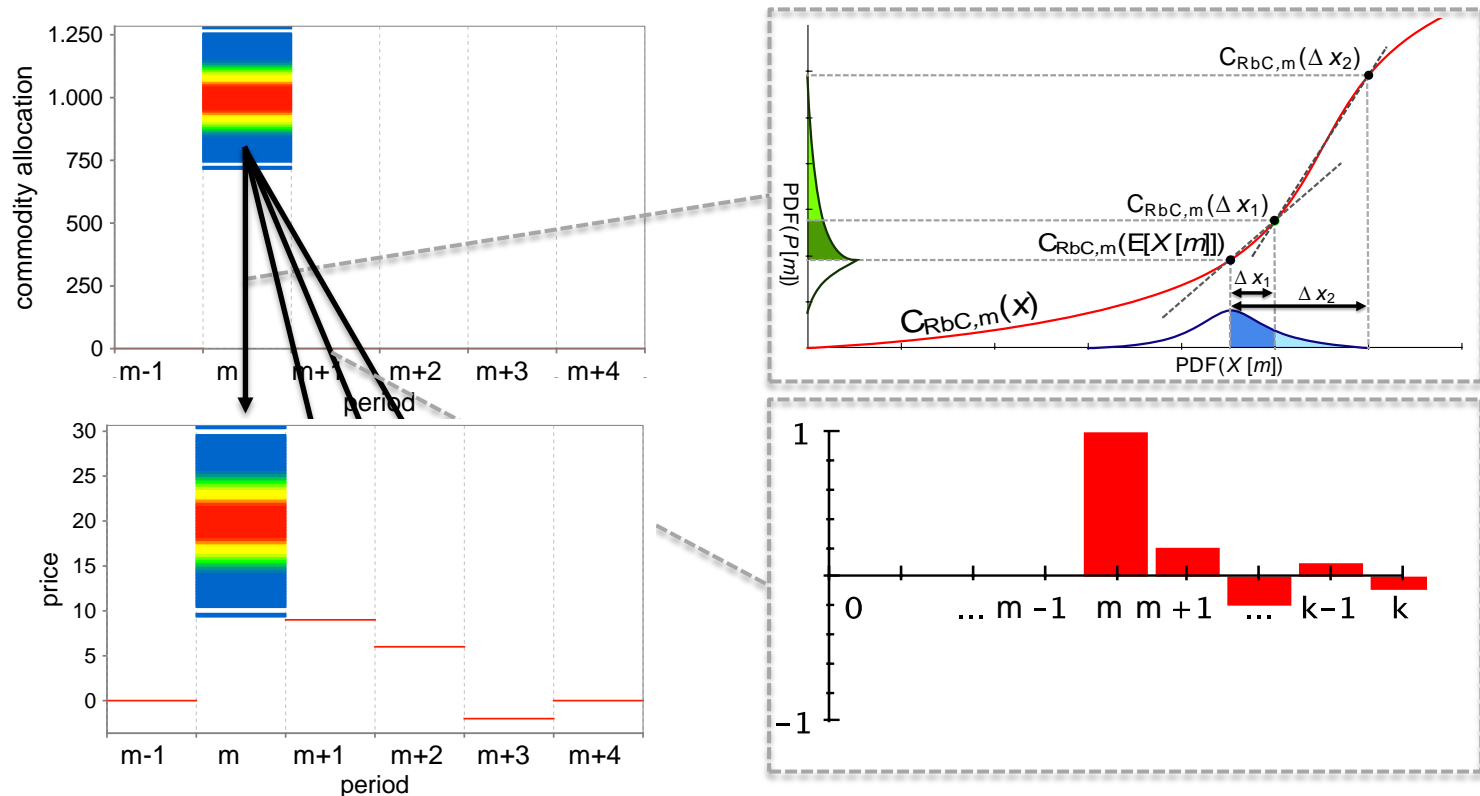
$$s.t. \quad \forall j < m : X'_i[j] = X_i[j]$$

$$P' := (P_1, \dots, P_{m-1}, P_m + \Delta p, P_{m+1}, \dots, P_k)^T$$



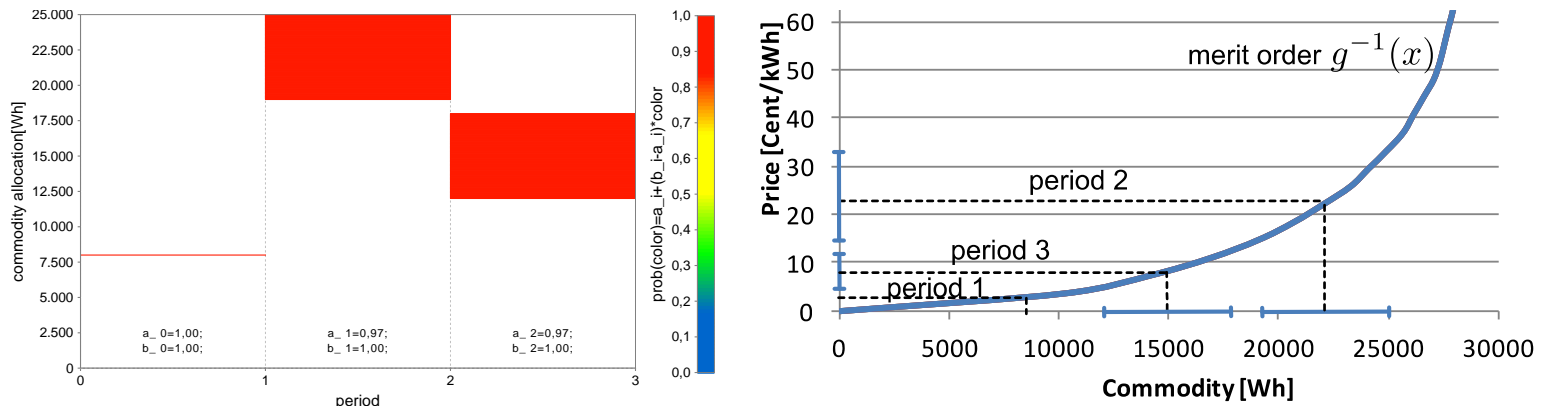
Mapping commodity uncertainties to price domain

- Given a well chosen set of Rebalancing Price Responses we are now able to approximate the merit order of all market participants.

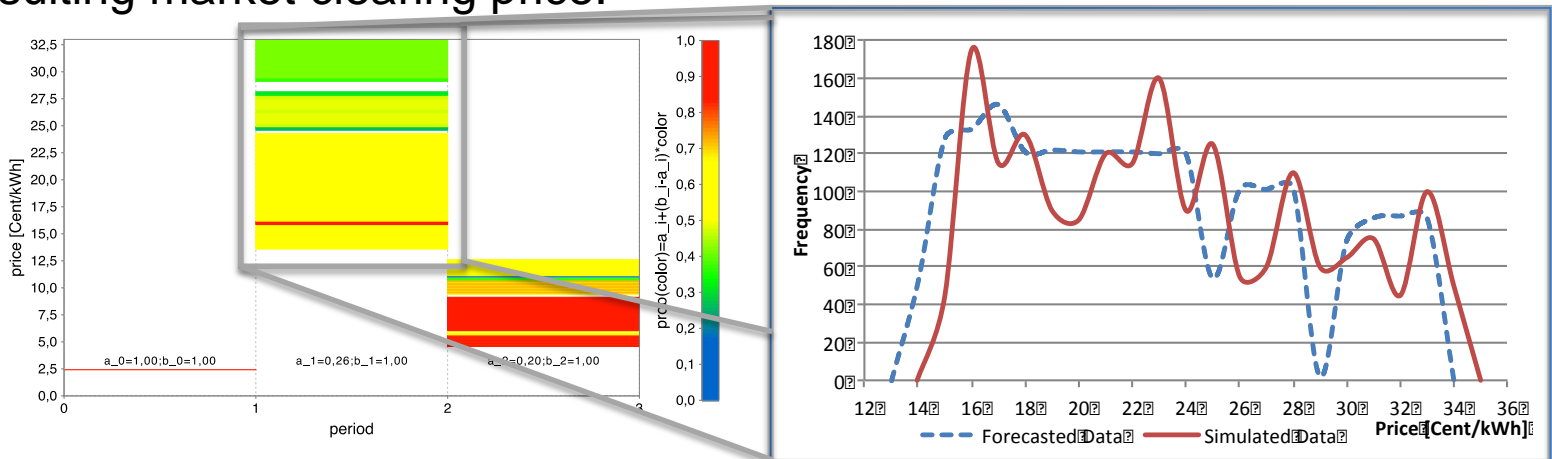


Results: Simple test scenario

- Market setup: stochastic demand (left) merit order of supply (right):

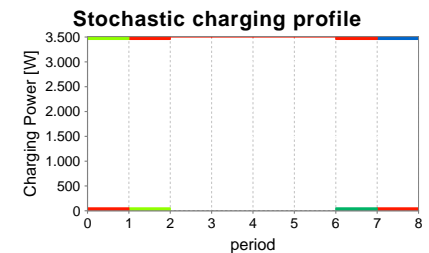
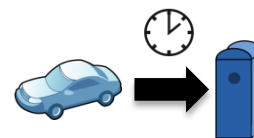
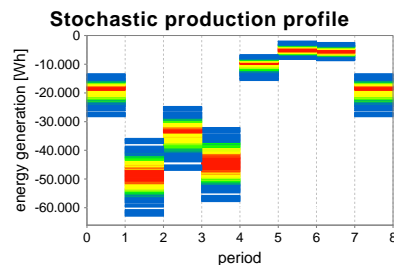
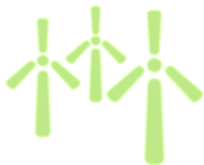


- Resulting market clearing price:



Expected future results

- Modeling of Grid operator → integration of grid constraints
- More robust unit commitment
- New incentives for demand response resources and storage devices
- Better integration of renewables, DG and DRR



Thank you for your attention!

